



**Fundamentals of  
Geophysical Fluid Dynamics**

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**CAMBRIDGE**

# FUNDAMENTALS OF GEOPHYSICAL FLUID DYNAMICS

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D III 296

Dk: 551.511

551.511.3

551.511.32

551.511.6

551.511.61

358/4273

INSTITUT  
FÜR METEOROLOGIE U. KLIMATOLOGIE

LEIBNIZ UNIVERSITÄT HANNOVER  
HERRENHÄUSER STR. 2 · 30419 HANNOVER



CAMBRIDGE  
UNIVERSITY PRESS

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